It may be added that the condition that (13) and (14) have the kind of contact consistent with the nature of the problem is

$$l_1^2(1 - e_2^2) + l_2^2(1 - e_1^2) - 2l_1l_2 + 2l_1l_2e_1e_2 \cos \gamma = 0. \tag{22}$$

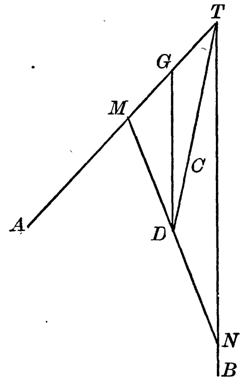
Also solved by HORACE L. OLSON.

2687 [March, 1918]. Proposed by N. P. PANDYA, Sojitra, India.

An ellipse intersects a parabola in A and B, and the tangents at A and B to the parabola meet at T. The center C of the ellipse lies within the space enclosed by the parabola and the tangents. Draw a third tangent to the parabola such that C may be the centroid of the triangle formed by the three tangents.

SOLUTION BY HORACE L. OLSON, Heidelberg University, Tiffin, Ohio.

This problem can be solved, if at all, without reference either to the ellipse or to the parabola. For this purpose let us alter the problem to read as follows: "Given two straight lines, TA and TB, intersecting at T, and a point C; draw a third line which shall form, with TA and TB, a triangle whose centroid shall be the point C." The problem, as thus stated, has a unique solution. If the third straight line is tangent to the parabola mentioned above, it is the solution of the original problem; if not, there is no solution. Draw the line TC, and extend it beyond C to D, so that $CD = \frac{1}{2}TC$. D will then be the mid-point of the third side of the required triangle. Through D, draw the line GD parallel to TB.



Lay off $GM = TG$. The line MDN, intersecting TB at N, will then be the required line; for, since the line GD is parallel to the side TN of the triangle TMN and bisects the side TM, it must also bisect the side MN. Since this demonstration is reversible, MN is the only line-segment included between the lines TA and TB, and bisected at D. Hence, if MN is tangent to the given parabola, it is the solution of the original problem; if not, there is no solution.

Also solved analytically by WILLIAM HOOVER.

2691 [April, 1918]. Proposed by ROGER A. JOHNSON, Hamline University.

Show by purely geometric methods, without the use of the calculus, that the envelope of the circles whose centers are on a fixed circle and which touch a fixed diameter of that circle is a two-arched epicycloid. Cf. problem 423 (calculus) [February and September, 1917].

SOLUTION BY THE PROPOSER.

First, let us make a few general remarks about the envelope of a system of circles. In general, the points where one of a system of curves meets the envelope of the system are the limiting positions of the points of intersection of that curve with a near-by one, as the latter comes into coincidence with the former. But two circles generally intersect in two points; the perpendicular bisector of the line connecting them is the line joining the centers of the circles. Hence,